An Improved Position Reconstruction Method For NSW sTGC

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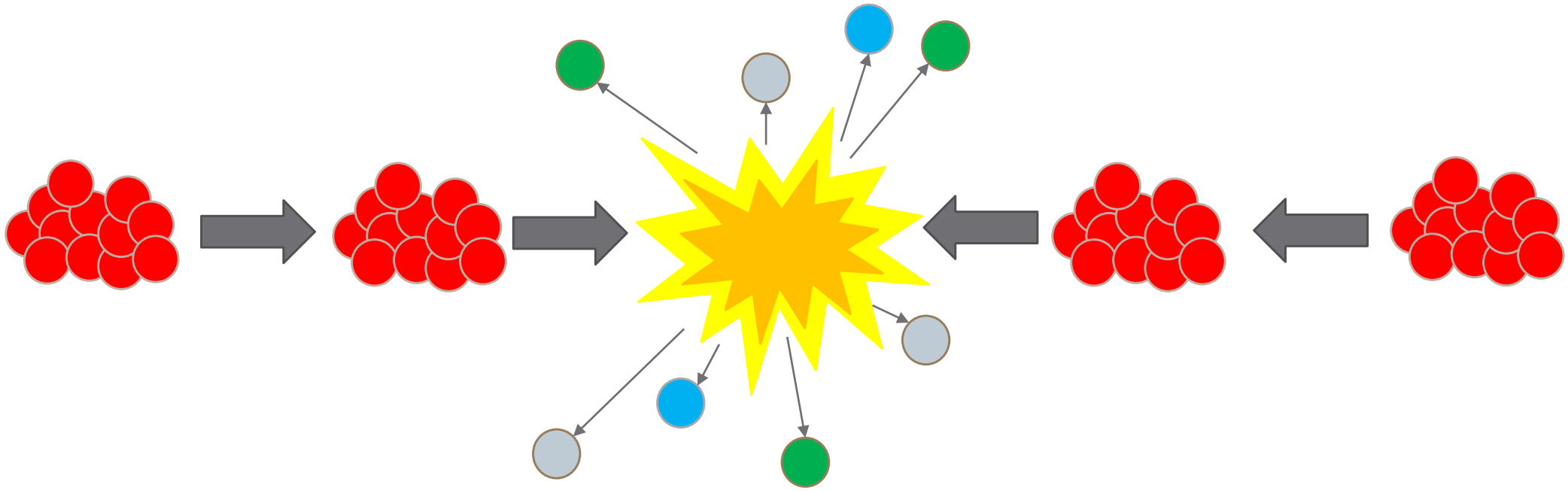
Outline

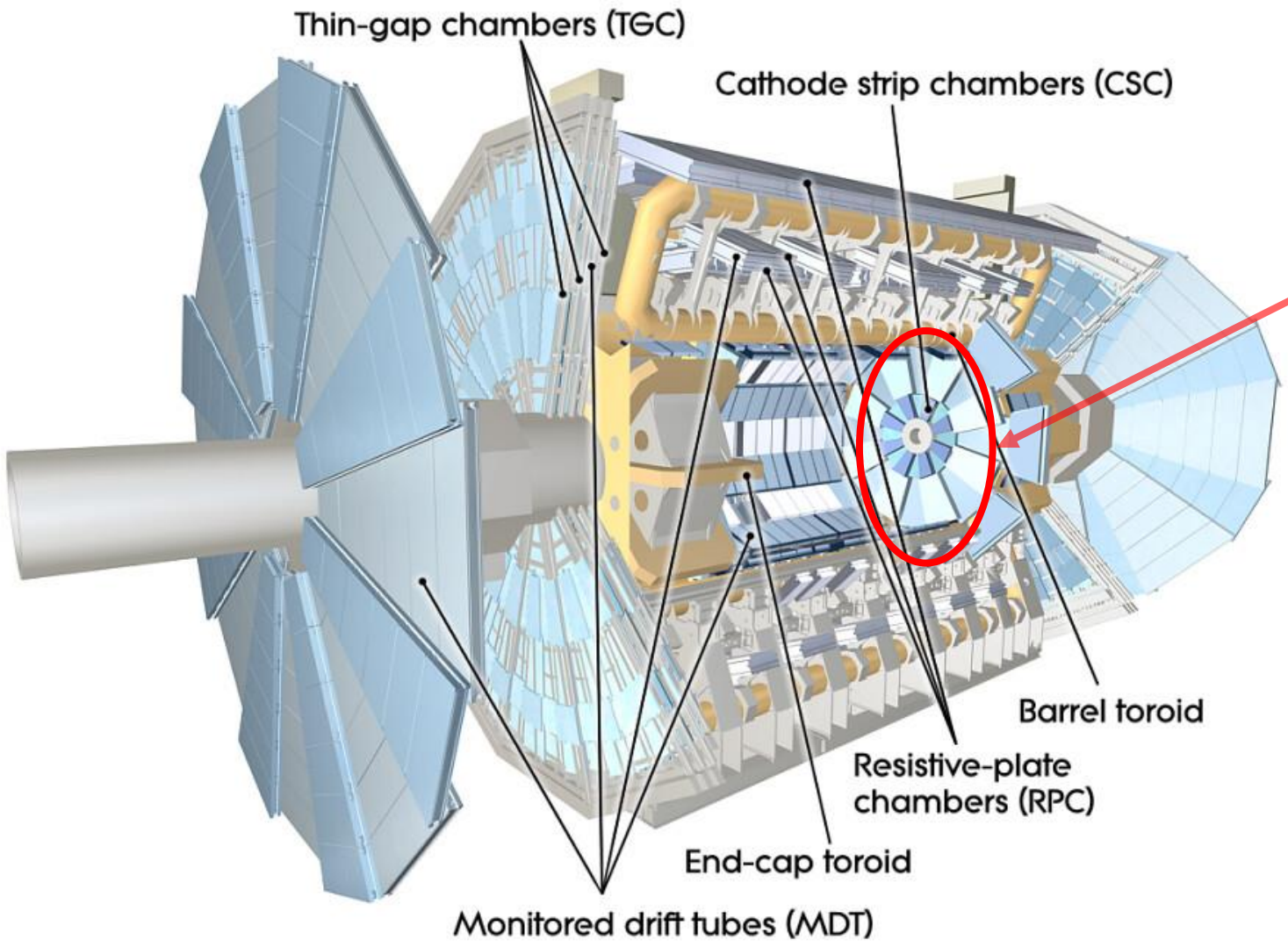
- Brief Background on ATLAS and the sTGC
- Current position reconstruction method
- New and improved position reconstruction method
- Reconstruction bias and comparison of reconstruction methods

Inside the LHC



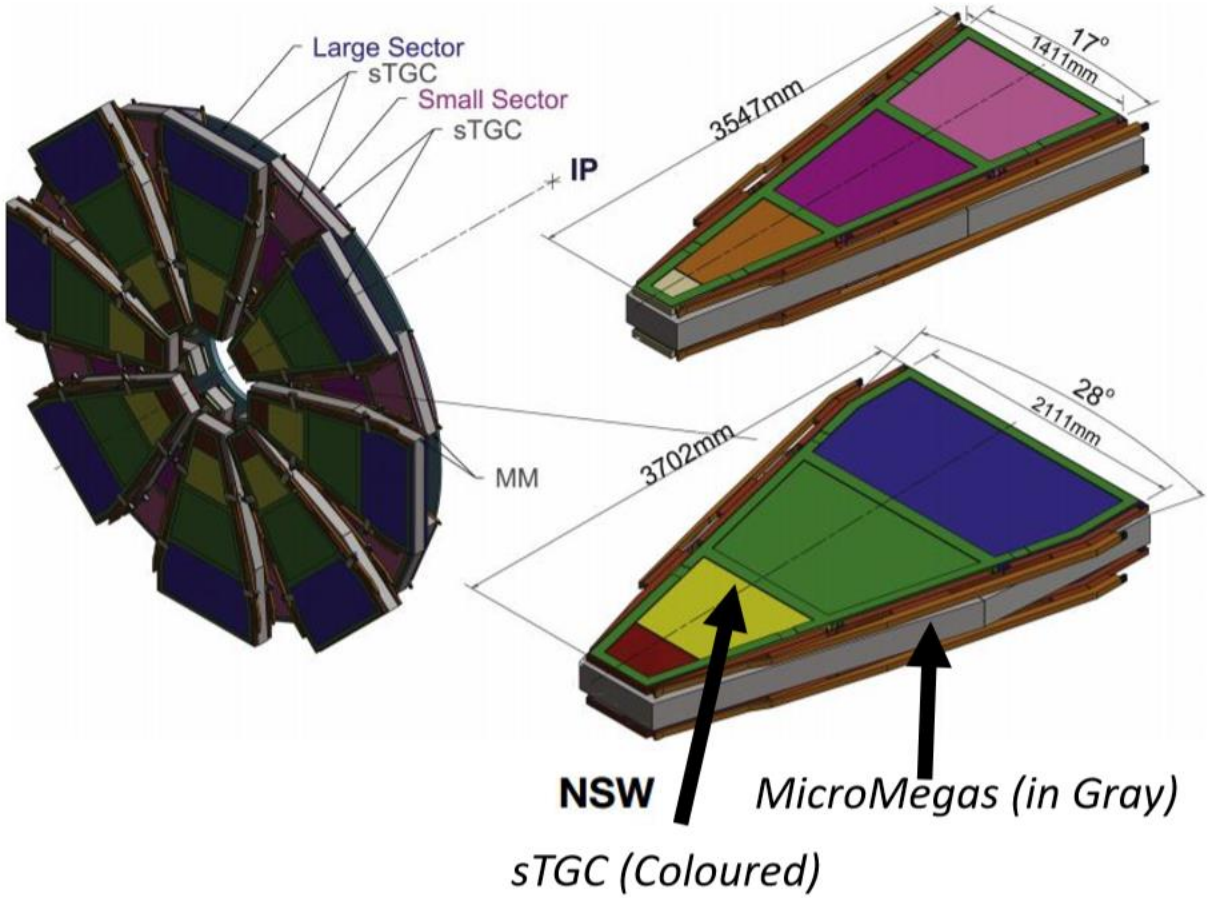
- During run 2, bunches of protons containing over 10^{11} protons per bunch are collided at a rate of 40MHz with a center point energy of 13TeV

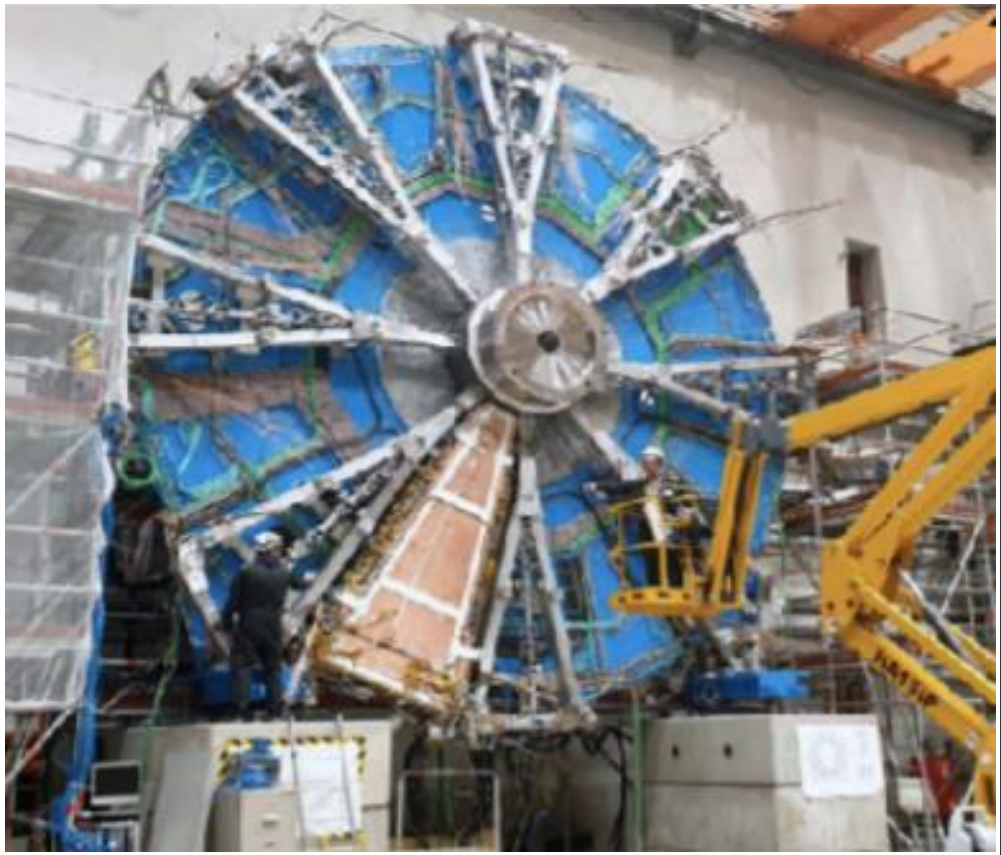
ATLAS Detector at the LHC



Current Small Wheel

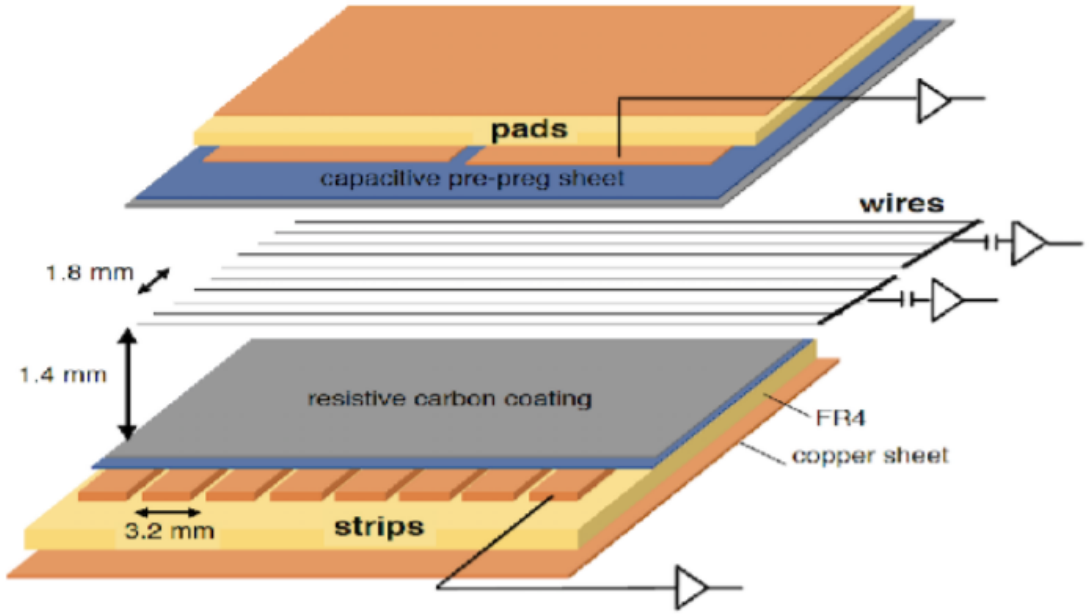
… which is now being replaced with the New Small Wheel

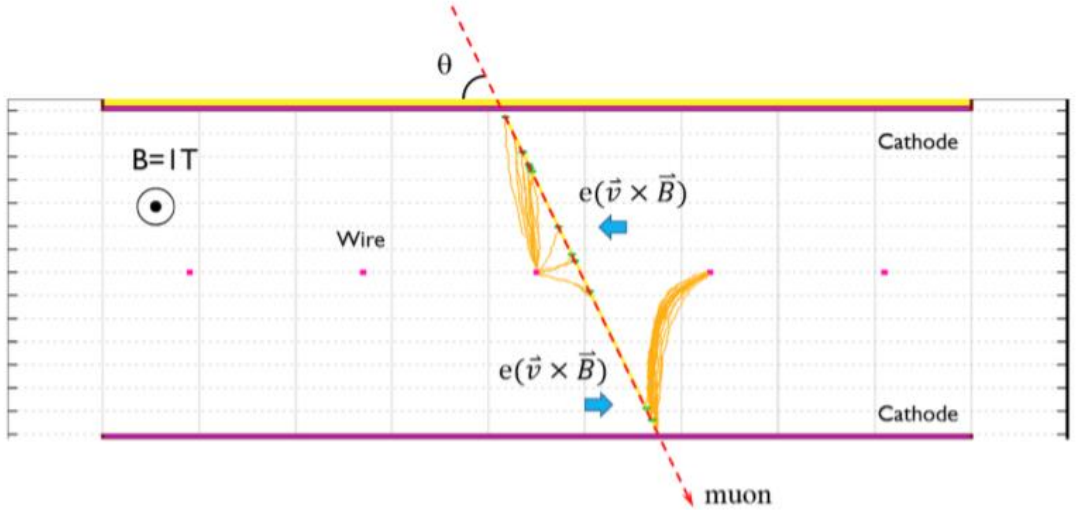




This together with the Big Wheel gives precision tracking of muons leaving the Interaction point (IP)

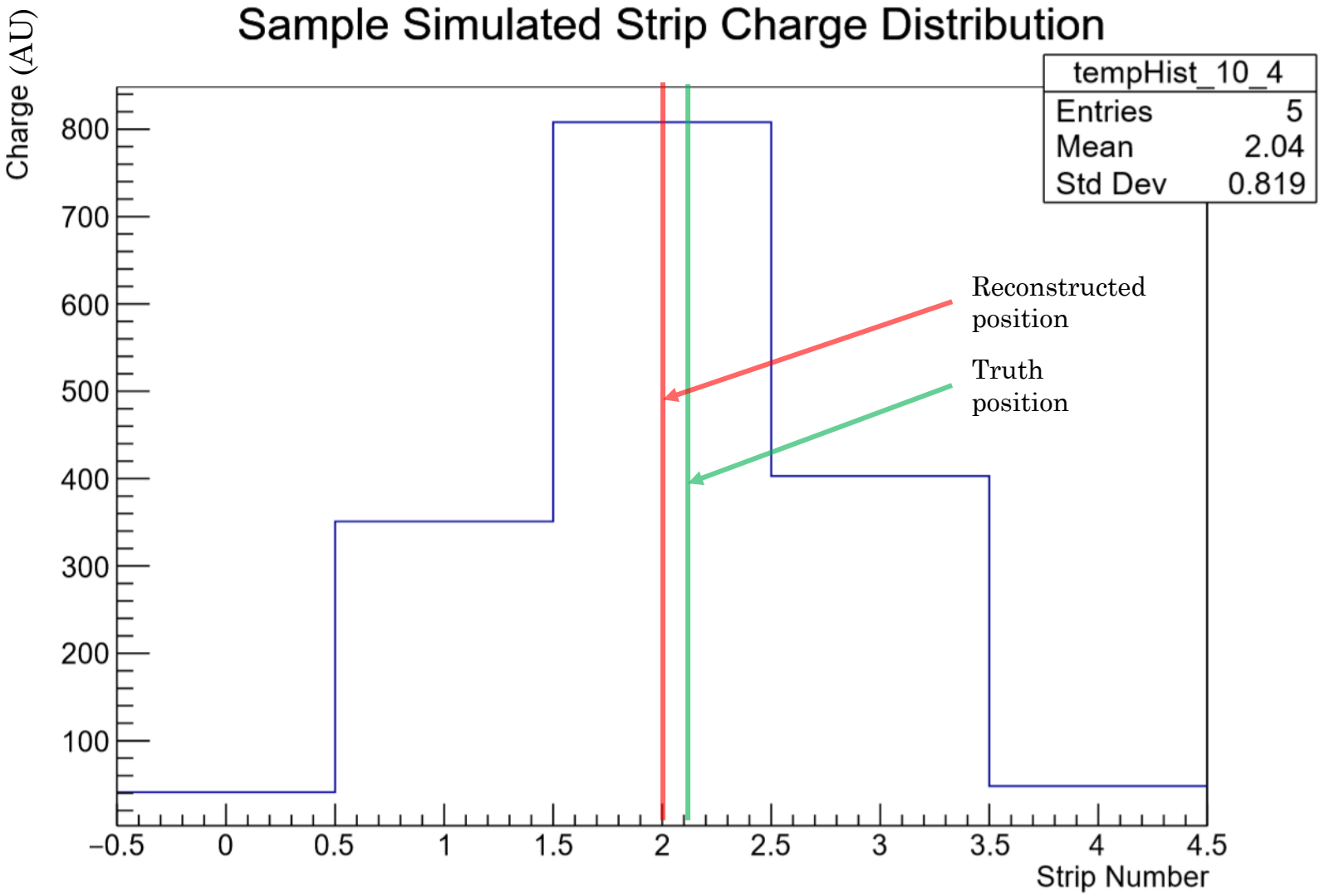
sTGC Design





- The NSW's sTGC layers are Multiwire Proportional chambers.
- Strong electric fields guide ionized electrons to sTGC wires (anode).
- Increasing field strength near wire causes an avalanche of electrons.
- Charge on wires induce charge on the resistive layer and strips.
- Induced charge in strips are used to precisely reconstruct the theta coordinate of the hit.

What Does a Hit Look Like?



- Left is a simulated hit that triggered a cluster of 5 strips
- The Residual is the difference between the reconstructed position and the truth position of the muon:

$$res = y_{reco} - y_{truth}$$

Current Reconstruction Method

The default reconstruction method uses a weighted mean estimator:

$$\hat{\mu} = \frac{\sum_{i=1}^{N} q_i x_i}{\sum_{i=1}^{N} q_i}$$

Where q_i and x_i is the charge measured and the position of the i^{th} strip respectively.

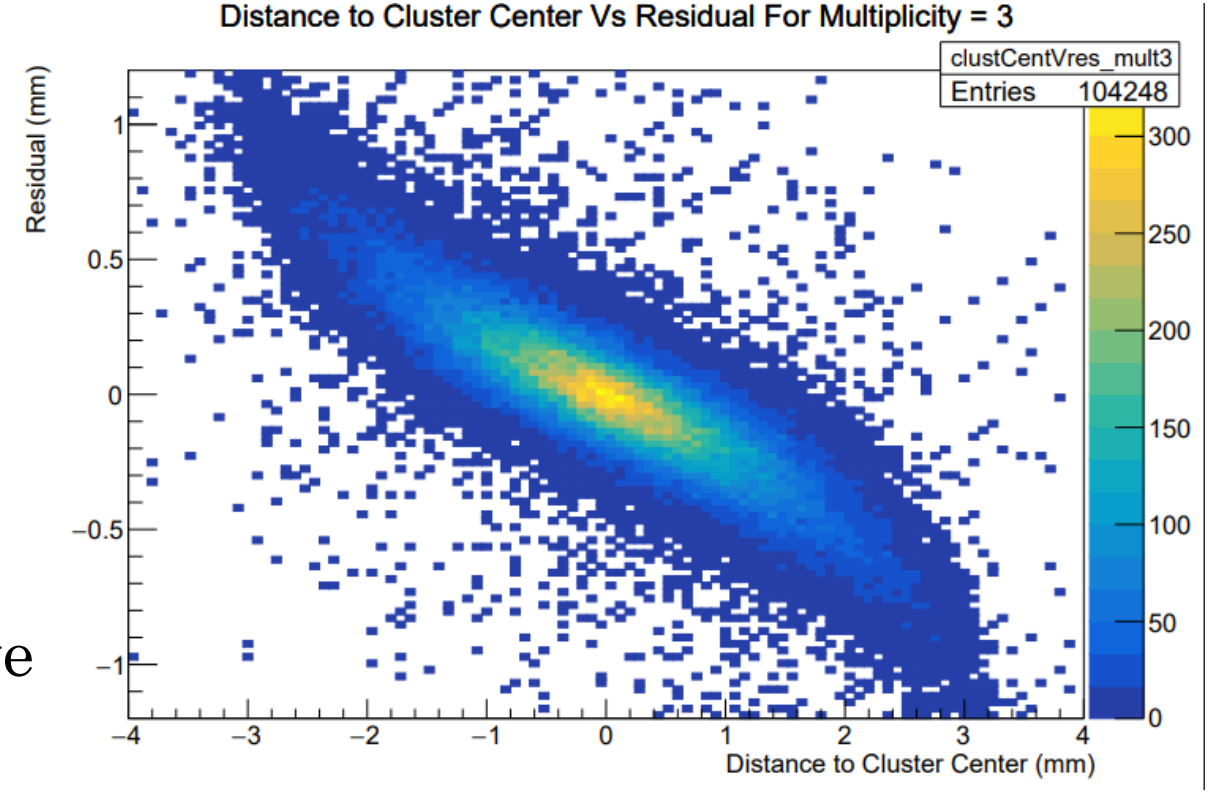
Estimator of the square of the error is given by:

$$\widehat{\sigma^2} = \rho^2 \frac{\sum_{i=1}^{N} q_i^2}{\left(\sum_{i=1}^{N} q_i\right)^2}$$

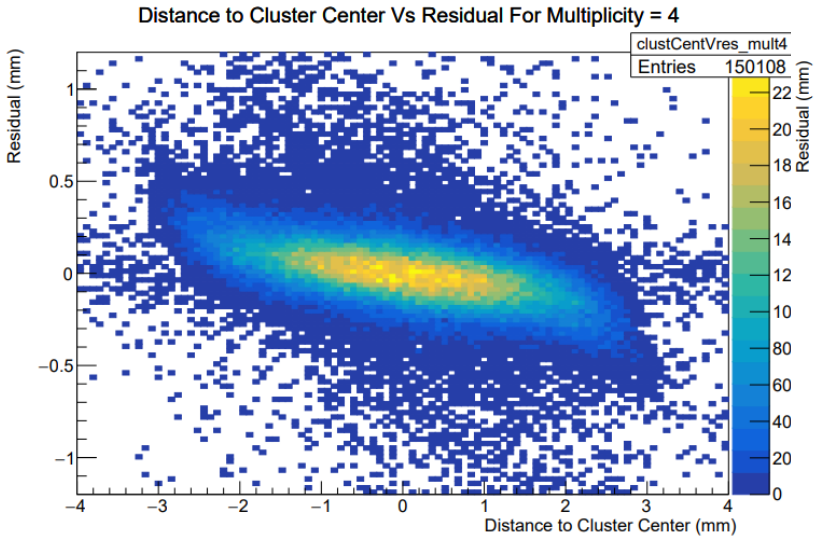
Where ρ is the single strip resolution.

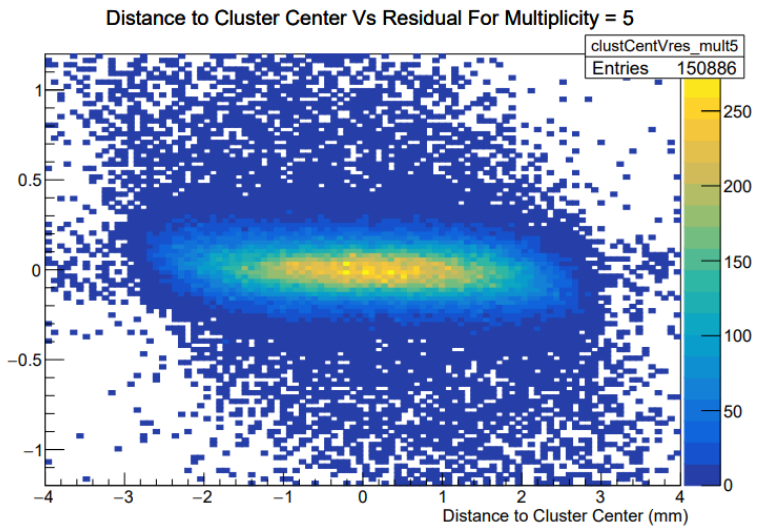
Problem (weighted mean)

- Multiplicity = Number of strips that register a charge above threshold
  (Vast majority of events have a multiplicity of 3, 4, or 5)

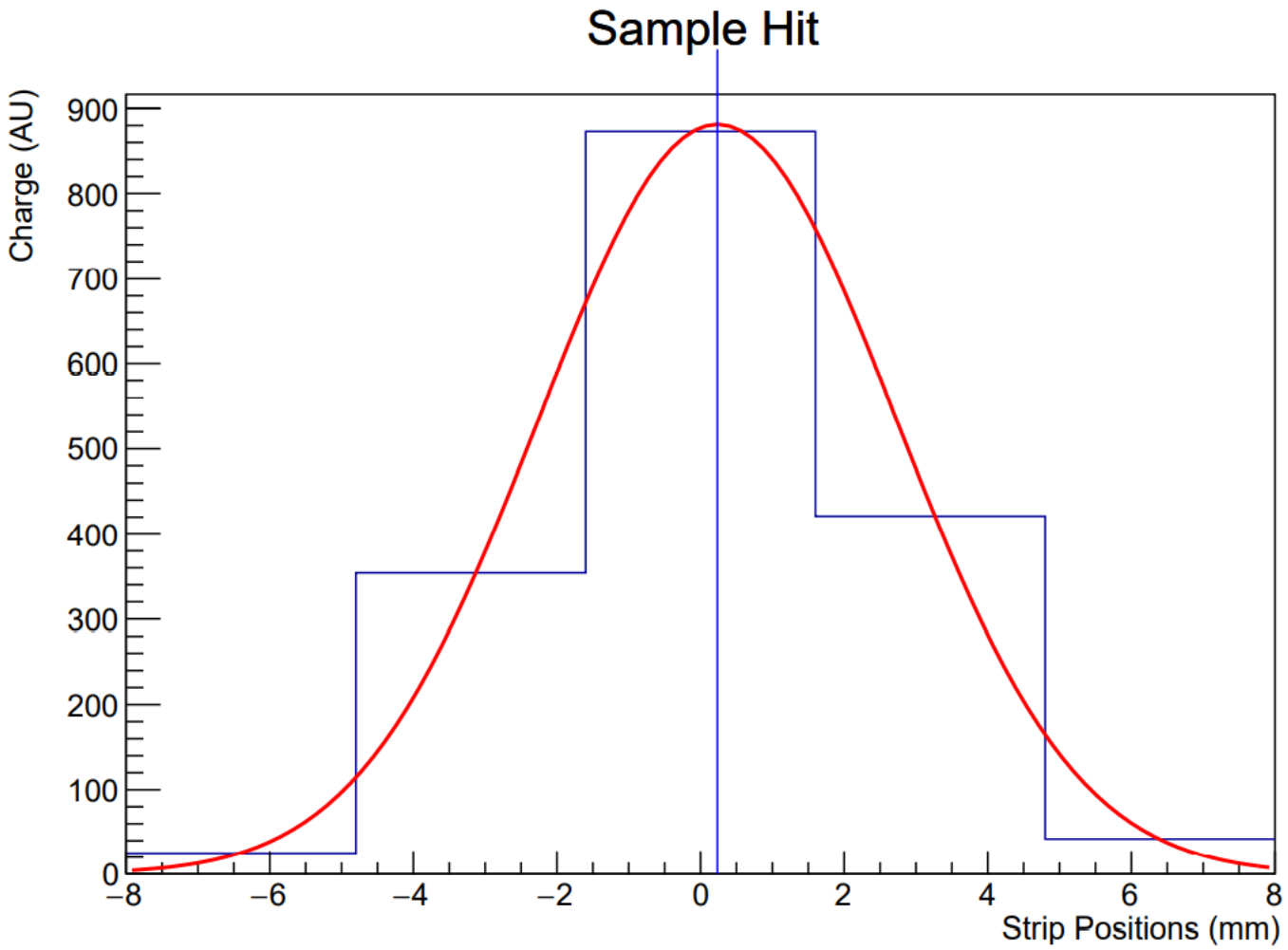


- Position estimator is biased with bias depending on multiplicity





New Position Reconstruction Method (Caruana Method)



- The charge induced on the strip plane can be approximated as a gaussian, hence we can instead try to fit a Gaussian to the measured charge distribution and use the mean as a position estimator

New Position Reconstruction Method (Caruana Method)

$$\ln\left(Ae^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) = \ln(A) - \frac{(x-\mu)^2}{2\sigma^2} = \ln(A) - \frac{\mu^2}{2\sigma^2} + \frac{\mu}{\sigma^2}x - \frac{1}{2\sigma^2}x^2$$

- Logarithm of a Gaussian is a quadratic. Can therefore fit a quadratic to the distribution $\{\ln(\hat{q}_i)\}$
- Using the properties of the sTGC, it can be shown that $\{\ln(\hat{q}_i)\}$ is distributed as

$$\ln(\hat{q}_i) = \ln(q(x_i)) + \eta$$

Where q(x) is a Gaussian function and η is a random variable whose distribution can be approximated as $Gaus(0, \nu)$ where ν only depends on the incident angle of the hit with respect to the sTGC surface.

New Position Reconstruction Method (Caruana Method)

- The ML fit then becomes a χ^2

$$\chi^2 = \sum_{i=1}^{N} \frac{(\ln(\hat{q}_i) - a - bx - cx^2)^2}{\nu^2}$$

- The parameters that minimize the above equation are the solution the matrix equation

$$\begin{bmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} \ln(\hat{q}_i) \\ \sum_{i=1}^{N} x_i \ln(\hat{q}_i) \\ \sum_{i=1}^{N} x_i^2 \ln(\hat{q}_i) \end{bmatrix}$$

- From this, $\hat{\mu} = \frac{-b}{2c}$

New Position Reconstruction Method (Caruana Method)

- Solving for the error on the mean, we find

$$\hat{\sigma}_{\mu} = \frac{\nu}{2c}\sqrt{\frac{1}{\gamma_2} + \frac{b^2}{c^2}\frac{\gamma_0}{\gamma_0\gamma_4 - \gamma_2^2}}$$

Where $\gamma_m = \sum_{i=1}^{N} x_i^m$ when the x_i are measured with respect to the center of the cluster

How To Compute the Reconstruction Bias

- Let x be the truth position of a hit relative to the center of the cluster, then the reconstructed position, y, is a function of truth position, hence, $y = y(x)$
- If h, is the PDF of the truth position, and f is the PDF of the reconstructed position, we require:

$$f(y)dy = h(x)dx$$

or,

$$\frac{dy}{dx} = \frac{h(x)}{f(y(x))}$$

How To Compute the Reconstructed Bias

This can be solved by the method of successive approximations. Assuming there is zero bias for hits with a truth position at zero (center of the cluster), then we define

$$\phi_n(x) = \int_0^x \frac{h(x')}{f(\phi_{n-1}(x'))}\,dx'$$

For $n \geq 1$ with $\phi_0(x) = 0$. Then,

$$y(x) = \lim_{n\to\infty} \phi_n(x)$$

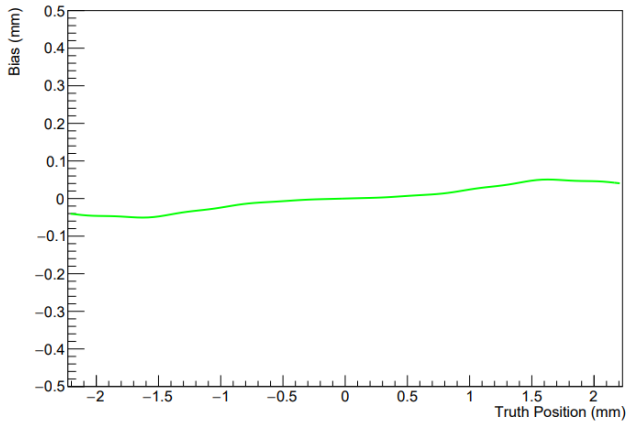
Method Comparison

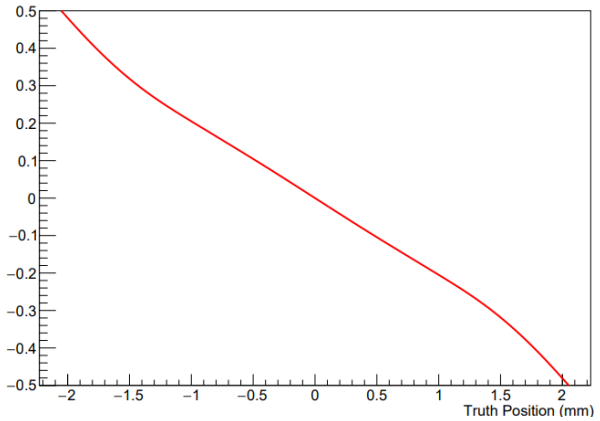
- Using a sample of simulated events we can solve for the reconstruction bias, getting the plots on the right.

Caruana Method

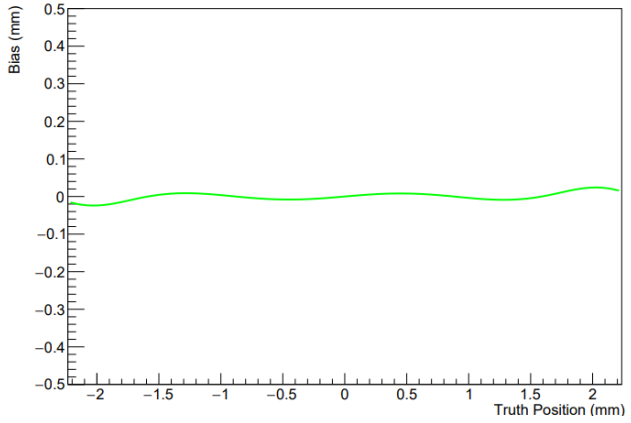
Weighted Mean Method

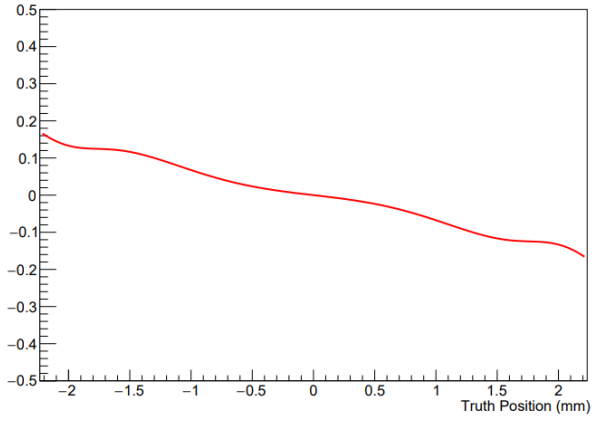
Multiplicity = 3



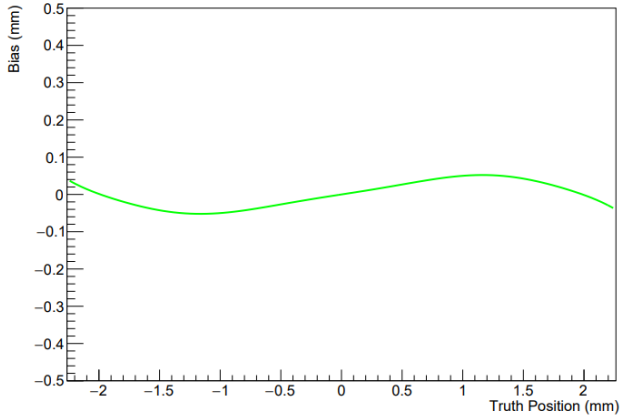


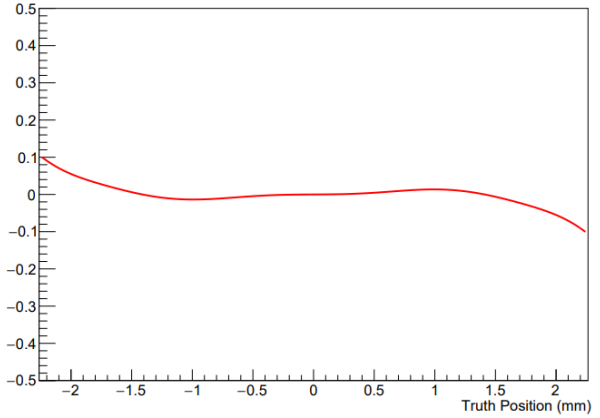
Multiplicity = 4



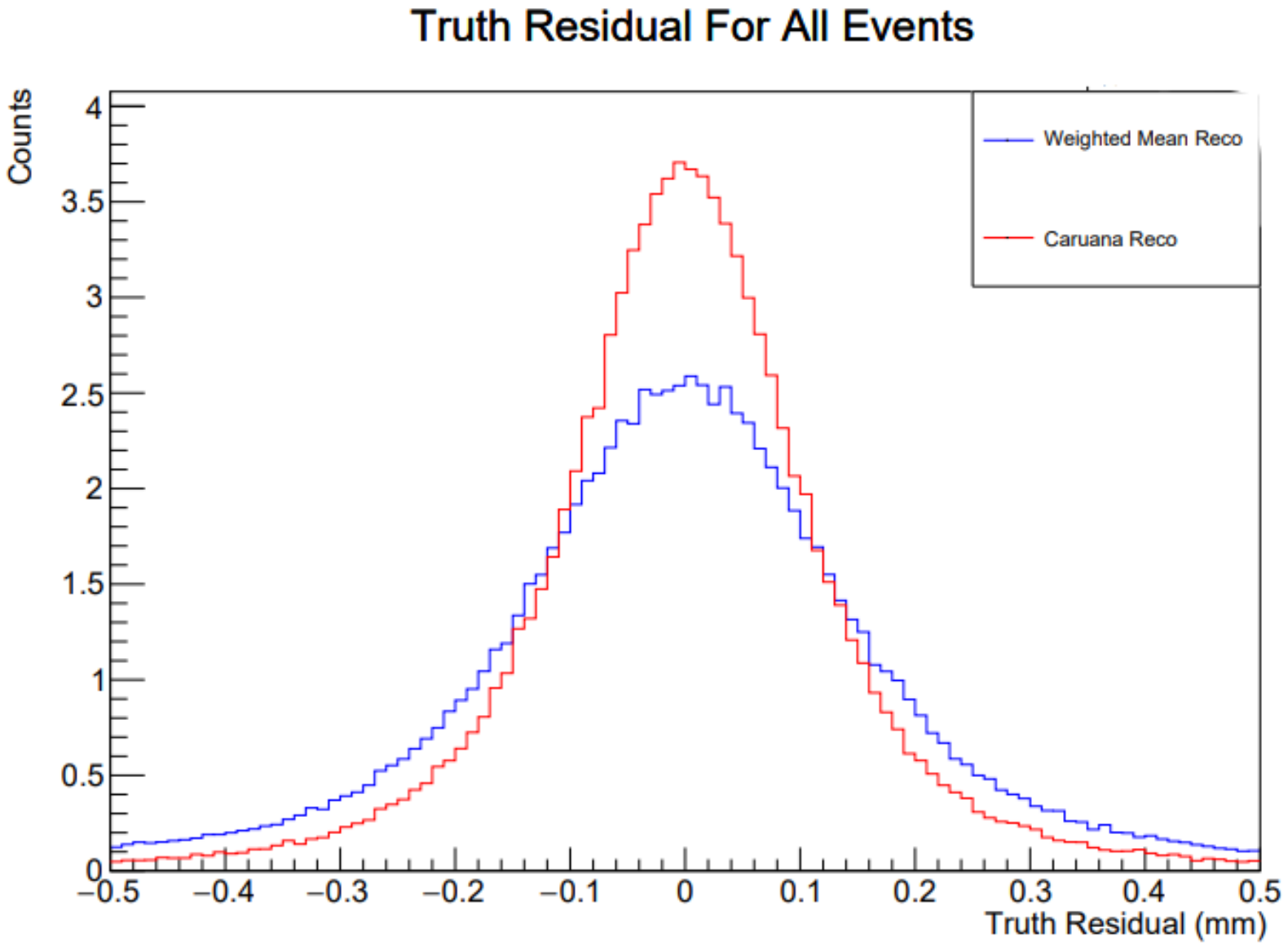


Multiplicity = 5

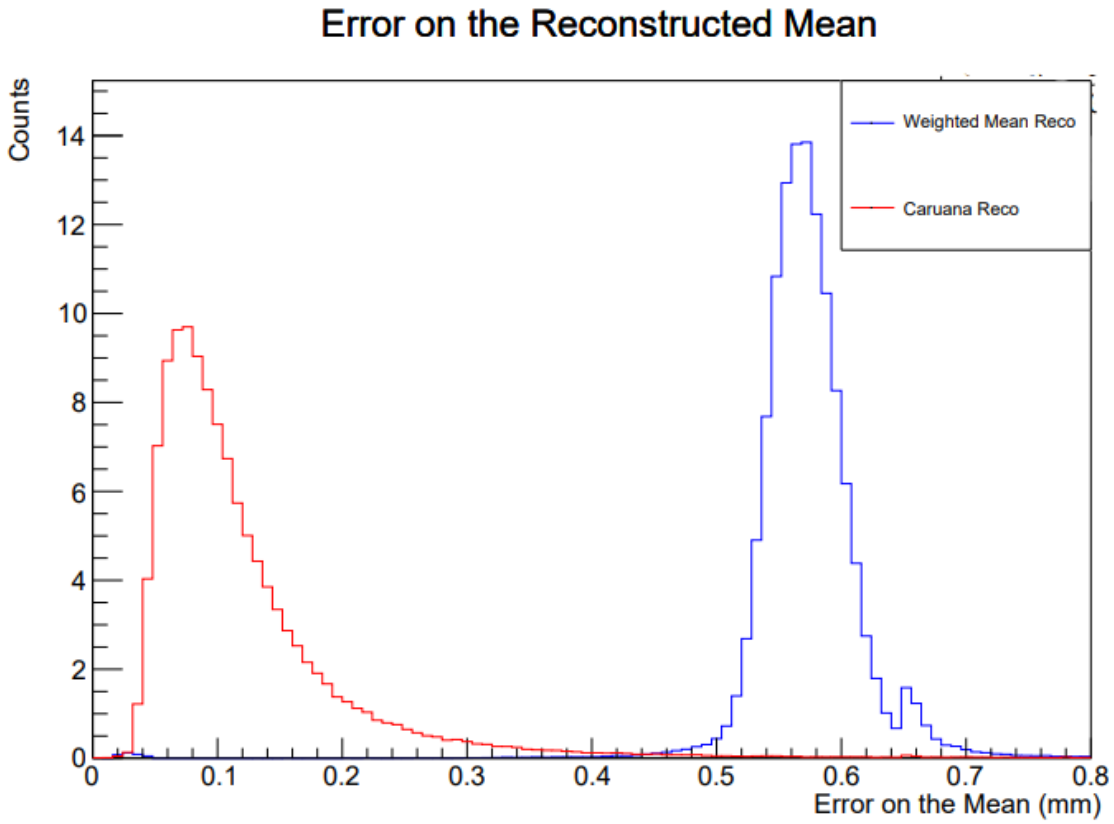


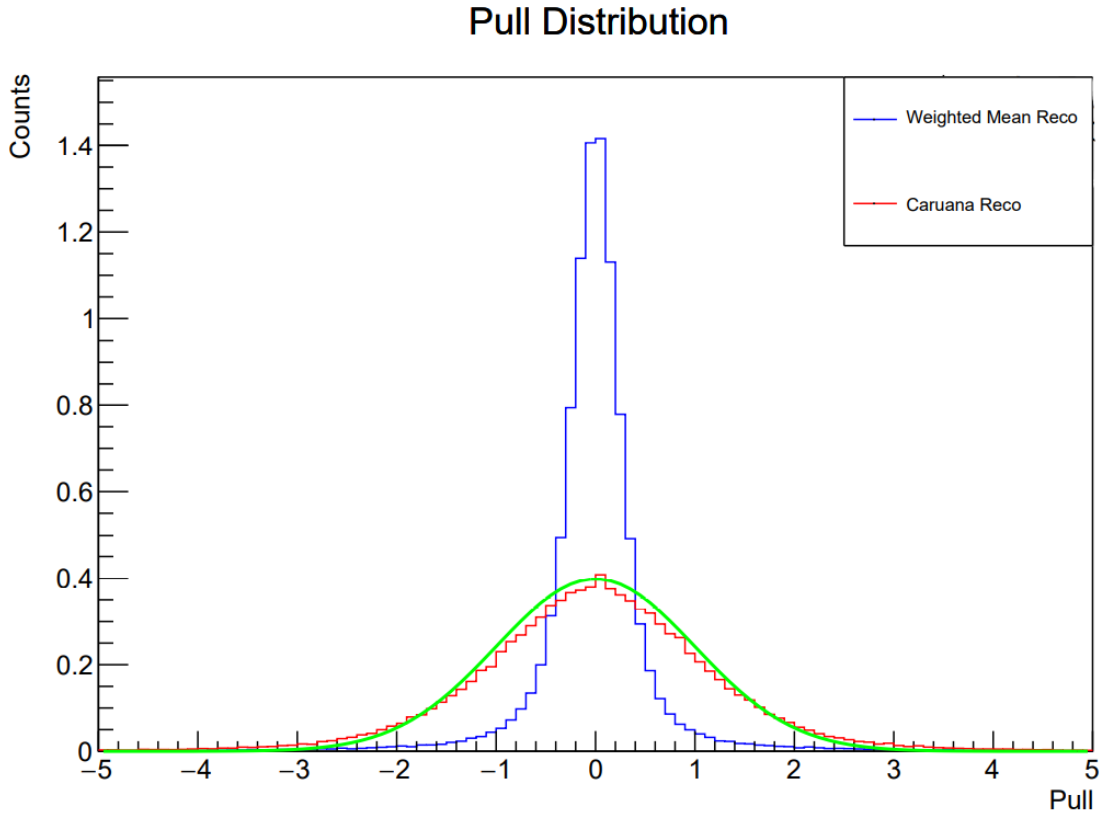


Method Comparison



Method Comparison





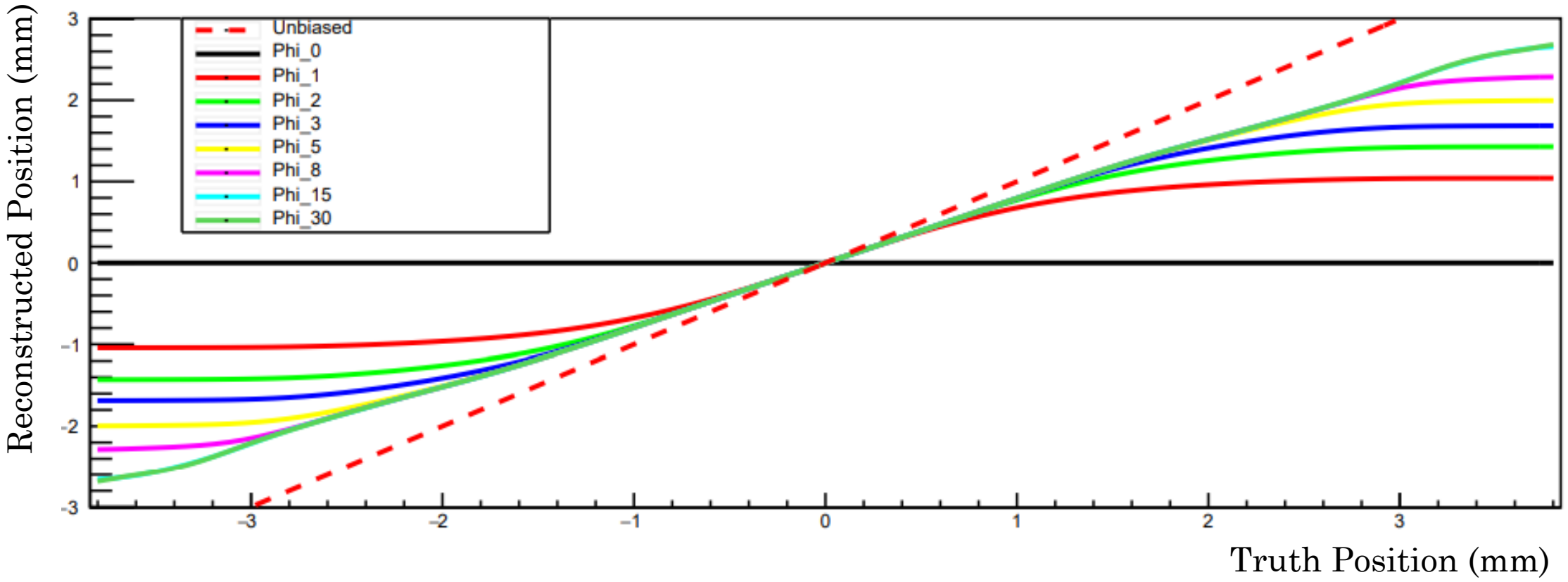
Pull Distribution of the new Caruana method is approximately N(0,1) which indicates that the estimator is unbiased and the error is computed correctly

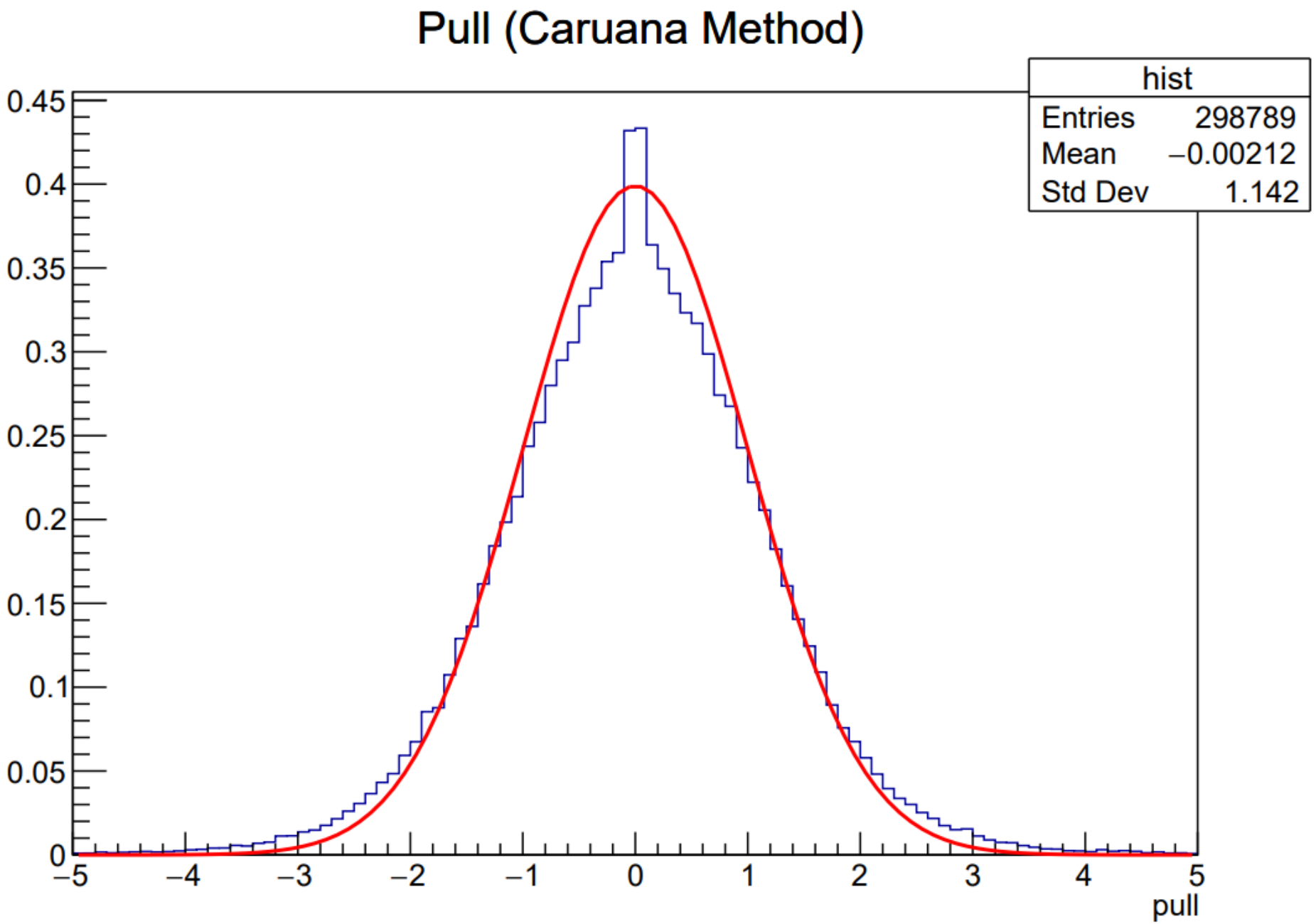
Summary

- Default sTGC hit reconstruction tool was observed to have a large bias
- An improved reconstruction method was developed based on the Caruana method of fitting a Gaussian
- The new method has been shown to be unbiased and has a better resolution.
- Reconstruction tool containing the Caruana method has been merged into ATHENA
- The Caruana method does not require any minimization at run time making it much faster than a fit using ROOT

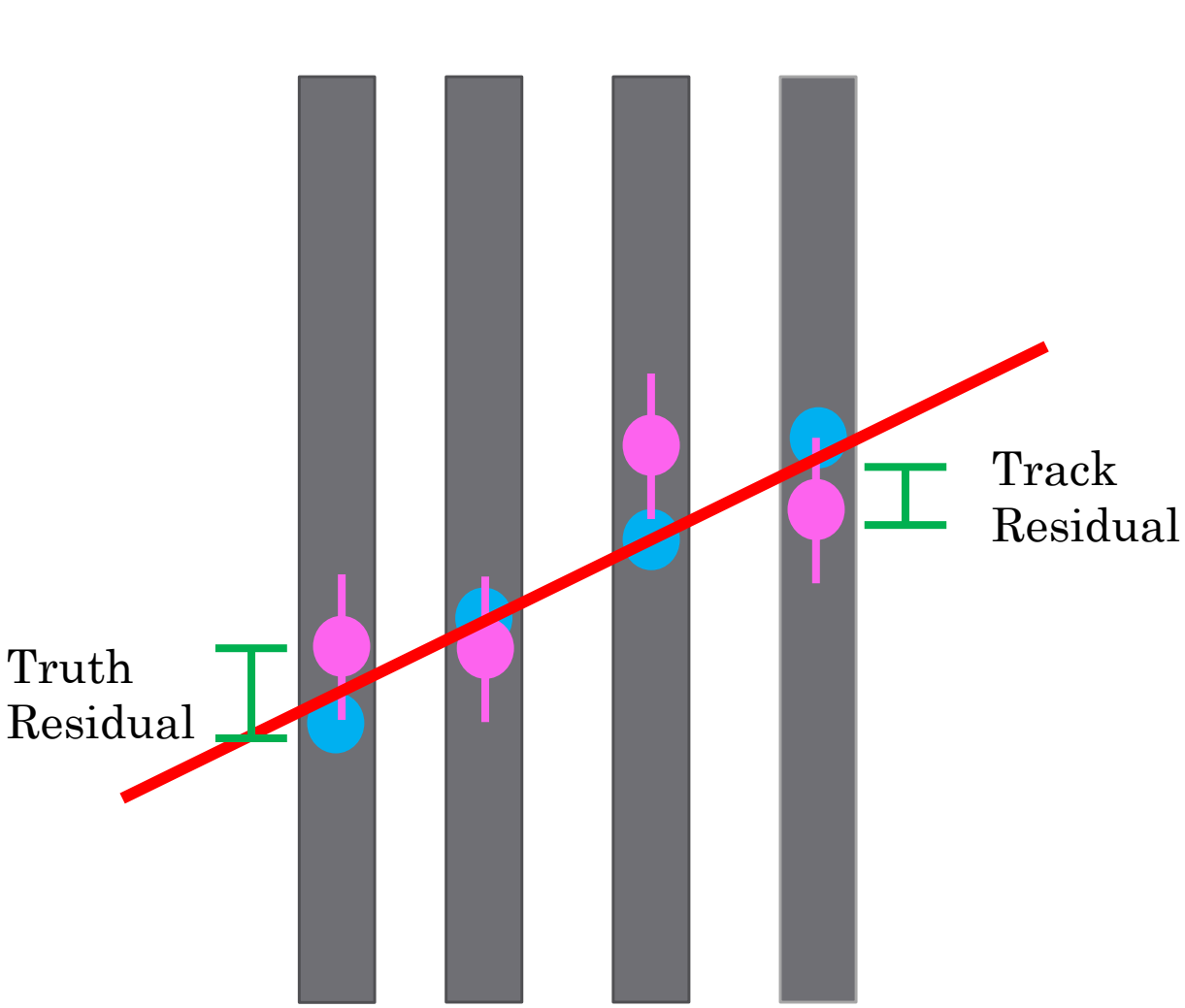
Backup Slides

Sample Computation of Reconstruction Bias For Weighted Mean Method With Multiplicity = 3




Pull (Caruana Method)
hist
Entries 298789
Mean −0.00212
Std Dev 1.142
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−5
−4
−3
−2
−1
0
1
2
3
4
5
pull

Truth Residual Vs Track Residual

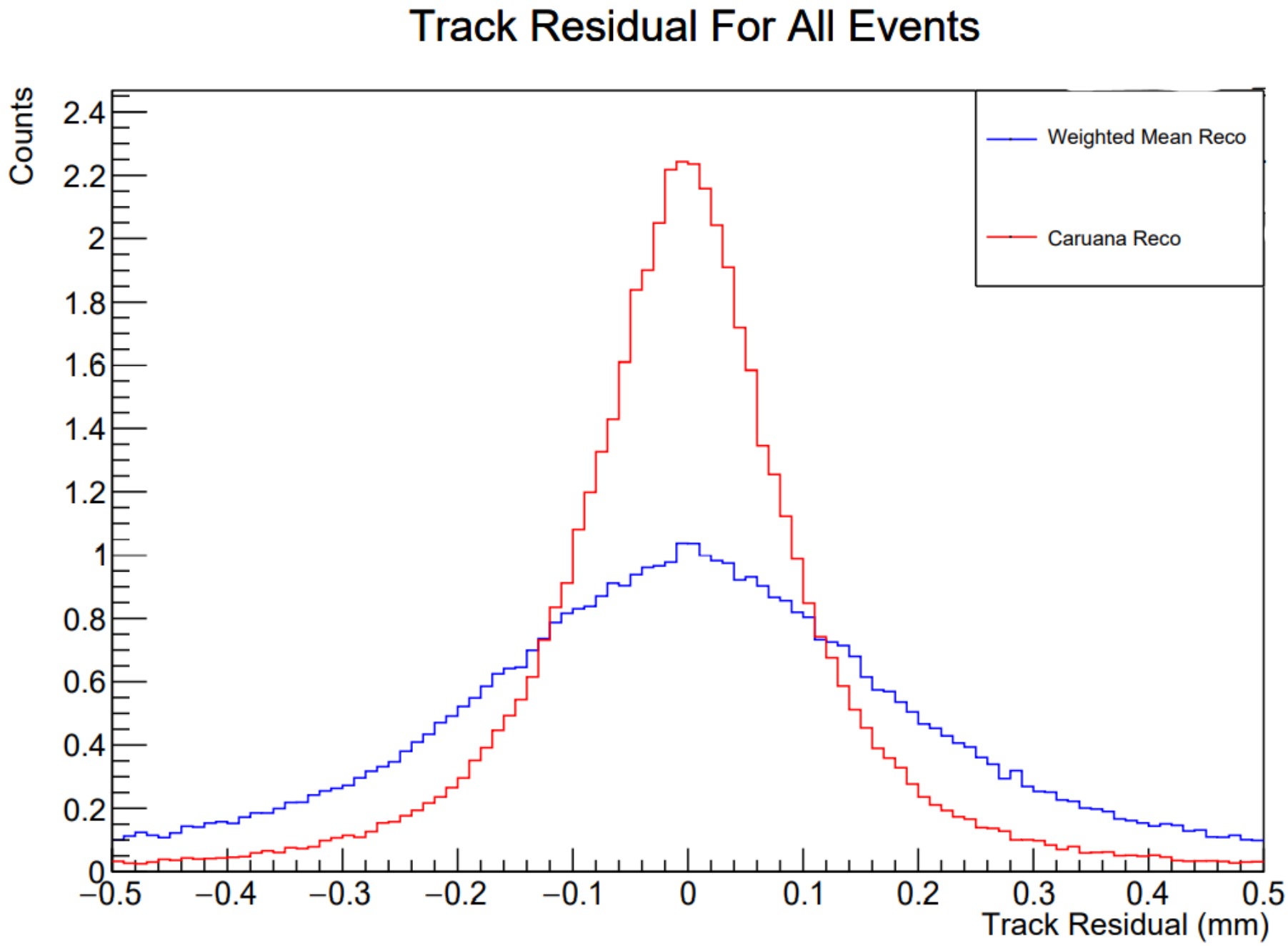


Reconstructed Position

Truth Position

Reconstructed Track

- Truth Residual – difference between the reconstructed position and the truth position.
- Track Residual – difference between the reconstructed position and the reconstructed track


Track Residual For All Events
Counts
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
Track Residual (mm)
Weighted Mean Reco
Caruana Reco